

Department Of Mathematics
 Indian Institute Of Technology Delhi
 MAL 390 – Statistical Methods and Algorithms
 Major Test – 01-05-2015

Time: Two Hour

Total Marks: 30

Q1. Consider the following bi-variate data table:

X	4	1	1	3	4	3	2	2	4	3	2	1	2	4	3	1
Y	1	1	4	1	4	2	4	3	3	4	2	3	1	2	3	2

- (a) Fit the two regression lines $Y = a + bX$ and $X = c + dY$ using the Least Square method and solving Normal equations.
- (b) What is the angle between the two lines? Justify your answer.
- (c) Use Least square theory to find the regression coefficients Y is regressed on $\alpha + \beta X$ for two non-zero constants α and β .

[3 + 1 + 2 = 6]

- Q2. (a) Explain the Wilcoxon Signed Rank test for comparing the central tendencies of two populations.
- (b) Explain how Kendall's Tau is used for computing the correlation between bi-variate observations.
- (c) Compute the value of Tau for the following data:

X	4	1	5	3	2
Y	1	3	4	2	5

[2 + 2 + 2 = 6]

- Q3. (a) Prove or Disprove: If T is the Maximum Likelihood estimator for θ , and Ψ is a strictly monotonic function, then $\Psi(T)$ is Maximum Likelihood estimator for $\Psi(\theta)$.
- (b) Prove or Disprove: If T is sufficient for θ , then T is also Maximum Likelihood estimator for θ .
- (c) Find the Maximum Likelihood estimate for $\frac{1}{\theta}$ from one observation on the random variable $X \sim \theta (1 - \theta)^{x-1}$, $x = 1, 2, 3, \dots$

[2 + 2 + 2 = 6]

Q4. (a) Distinguish between Most Powerful and Uniformly Most Powerful Critical regions while testing a statistical hypothesis.

(b) Use Neyman-Pearson Lemma to construct the Most Powerful Critical Region for testing $H_0 : \theta = \theta_0$ vs. $H_1 : \theta = \theta_1$ where $\theta_0 < \theta_1$, based on a sample of size n taken from $N(0, \theta)$, at 95% level of confidence. Note: θ stands for the variance.

[2 + 4 = 6]

Q5. (a) Describe the Lehmer's algorithm for generating random numbers.

(b) Suppose the following 10 random numbers are generated in the range 1 - 100:

15, 92, 81, 23, 18, 73, 64, 95, 41 and 32.

Use the above numbers to generate 10 random numbers from $N(0,1)$ population. Justify your answer.

[3 + 3 = 6]